

Name:

School: Middle School

Grade: 10th

I. **Subject Area:** Mathematics

II. **Topic and Core Standard:** Geometry

Standard: 4.2A.2 Understand and apply the Pythagorean theorem.

III. **The Concept:** The Pythagorean Theorem

IV. **Concept Statement:** In a right angled triangle the square of the long side (hypotenuse) is equal to the sum of the squares of the other two sides (legs).
 $a^2 + b^2 = c^2$. This is the Pythagorean Theorem.

V. **Objectives/Student Learning Outcomes:**

Students will be able to:

Low Level Objectives:

1. Define the Pythagorean Theorem
2. Understand terminology associated with the Pythagorean Theorem, such as leg, hypotenuse, opposite, adjacent, and Pythagorean triple.
3. Construct the three measures of right triangles using the Pythagorean Theorem.

High Level Objectives:

1. Apply the Pythagorean Theorem to real world problems.
2. Generate Pythagorean triples.
3. Judge the importance of the Pythagorean Theorem.

VI. **Teacher Actions**

1. **Planning:**

Prior Knowledge: Students will need to have prior knowledge of the three different types of triangles. They will have to know there is a right triangle, an obtuse triangle, and an acute triangle. If necessary we will go over the definitions of these types of triangles with emphasis on right triangles.

Seating Arrangement: Students will sit in groups of two or three.

Grouping: Students will be grouped heterogeneously by ability.

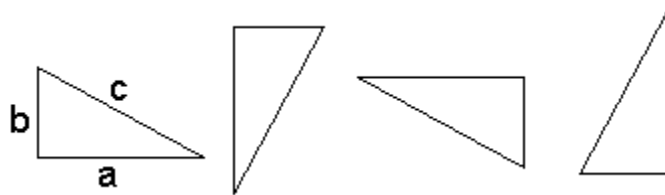
Materials/Resources: The materials that we will need are: smart board, textbook, triangles, and computers.
(Textbook: Algebra 1 Glencoe McGraw-Hill. By: Holliday, Luchin, Marks, Day, Cuevas, Carter, Casey, and Hayek. Copyright 2008 by the McGraw-Hill Companies, Inc.)

2. **Motivate:** The teacher will hand out four triangular tangrams to the students. The teacher will ask them to take the tangrams and create a square. Next, the teacher will ask them to create two squares. Then, take one triangle and ask the students to rotate it 90 degrees, 180 degrees, and 270 degrees. The teacher will tell the students to make one big square and one little square with the triangle facing in these different directions. Explain that there should be a square within a square and the space does not have to be closed. Give the students a few minutes to discover this figure.

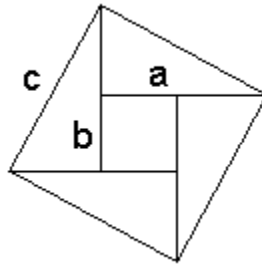
3. **Teach:**

- The teacher will discuss Pythagoras and the Pythagorean society as background information.
 - i. Pythagoras was a great mathematician that developed the Pythagoras theorem. Pythagoras was also part of the Pythagorean society. They believed that there were three different kinds of people. Ones that came to buy and sell, ones who came to compete, and others that just came to look on. This society had many beliefs. One was that men and women were equal. Another was they all enjoyed the common ways of life. Third was that property was communal. Last, all mathematical discoveries were communal. This society was very strict and believed all discoveries had to be credited to the whole society. They even took oaths to abide by this belief. This is where a controversy came about in regards to the Pythagorean Theorem. The society didn't want Pythagoras Theorem to be released because it crushed their original beliefs. As we know the theorem was released and is very well known today. After the release of Pythagoras's theorem, it is not really known what happened to him. The society was very disappointed.

- The teacher will review right triangles, obtuse triangles, and acute triangles and remind students a right triangle is a triangle that contains a right angle.
- The teacher will show the students how to derive one of the many proofs of the Pythagorean theorem from the tangram example. The teacher will show them how the triangles creating a square can derive the equation $a^2 + b^2 = c^2$.



Now we start with four copies of the same triangle. Three of these have been rotated 90° , 180° , and 270° , respectively. Each has area $\frac{ab}{2}$. Let's put them together without additional rotations so that they form a square with side c .



The square has a square hole with the side $(a - b)$. Summing up its area $(a - b)^2$ and $2ab$, the area of the four triangles $(4 \cdot \frac{ab}{2})$, we get

$$\begin{aligned}
 c^2 &= (a - b)^2 + 2ab \\
 &= a^2 - 2ab + b^2 + 2ab \\
 &= a^2 + b^2
 \end{aligned}$$

- The teacher will discuss the definition of the Pythagorean theorem, leg, and hypotenuse.
 - i. Pythagorean Theorem – If a and b are the lengths of the legs of a right triangle c is the length of the hypotenuse, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.
 - ii. Leg – In a right triangle, the side opposite the right angle is the hypotenuses, the other two sides are legs.
 - iii. Hypotenuse –In a right triangle, the side opposite the right angle.

- The teacher will introduce the Pythagorean triple, then show examples.
 - i. Pythagorean Triple – A group of three whole numbers that satisfy the Pythagorean Theorem.
 - ii. Examples include groups such as (3,4,5), (5, 12, 13), and (6, 8 ,10)

- The teacher will go over examples of solving triangles given one leg and one hypotenuse, two legs. Then go over real world examples using the same concepts.

Example 1) Find the length of the hypotenuse of a right triangle if $a=8$ and $b=15$.

Example 2) Find the hypotenuse of the triangle.
 $a=7$ and $b=12$

Example 3) Find the missing side.
 $a=10$, $b=?$, $c=25$

Example 4) Find the missing side.
 $a=?$, $b=6$, $c=14$

Example 5) A square shopping center has a diagonal walkway form one corner to another. If the walkway is about 70 meters long, what is the approximate length of each side of the center?

- A) 8 meters
- B) 35 meters
- C) 50 meters
- D) 100 meters

- The teacher will have the students work on examples by themselves.

Example 1) Find the hypotenuse

$a=10$, $b=6$ $c=?$

Example 2) Find the length of the missing side.

$a=?$, $b=11$, $c=21$

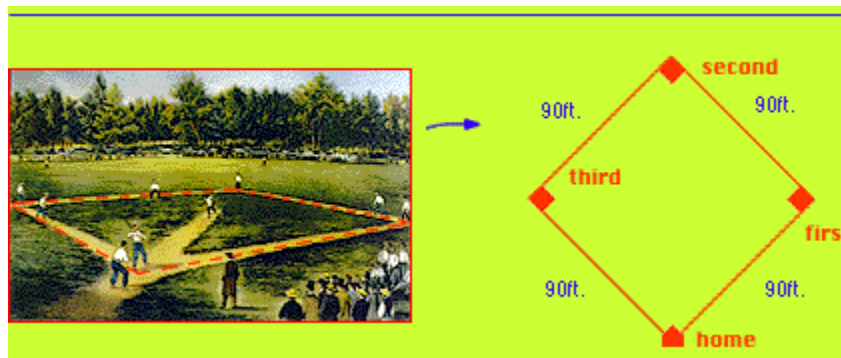
Example 3) The door of Julio's apartment measures 7 feet high and 3 feet wide. Julio would like to buy a square table that is 7 feet on a side. If the table cannot go through the door sideways, will it fit diagonally? Explain.

- The teacher will ask the students what is the objective of the Pythagorean Triple. If they don't answer, the following will be talked about in class discussion.
 - i. It allows us to solve theorem with whole numbers.
 - ii. It makes measuring things easier because there aren't decimals.
 - iii. It makes all real world applications easier because it does not involve decimals.
- The teacher will ask the students to judge the importance of the Pythagorean theorem. The students will be asked to explain why or why not they think the Pythagorean theorem is important. The teacher will ask the students their input first then have a class discussion. If the students don't answer, the following will be discussed.
 - i. The Pythagorean theorem can be used in your house, outside, in jobs etc.
 - ii. The Pythagorean theorem can be used in the following jobs.
 - Computer and information systems managers
 - Construction managers
 - Engineering and natural sciences managers
 - Computer software engineers
 - Mathematicians
 - Architects, except landscape and naval
 - Landscape architects
 - Surveyors, cartographers, photogrammetrists, and surveying technicians
 - Aerospace engineers
 - Chemical engineers
 - Civil engineers
 - Computer hardware engineers
 - Electrical and electronics engineers, except computer
 - Environmental engineers
 - Industrial engineers, including health and safety
 - Materials engineers
 - Mechanical engineers
 - Nuclear engineers
 - Drafters
 - Biological scientists
 - Conservation scientists and foresters

Atmospheric scientists
Chemists and materials scientists
Environmental scientists and geoscientists
Physicists and astronomers
Lawyers
Archivists, curators, and museum technicians
Writers and editors
Optometrists
Physicians and surgeons
Veterinarians
Opticians, dispensing

4. **Image:**

You've just picked up a ground ball at first base, and you see the other team's player running towards third base. How far do you have to throw the ball to get it from first base to third base, and throw the runner out?



$$\text{Answer} = 90^2 + 90^2 = c^2 = 127.28$$

5. **Ask lower and higher level questions**

Low

- What are the different types of triangles?
- State the Pythagorean Theorem.
- What is the hypotenuse of a triangle? What is the leg of a triangle?
- If two legs of a triangle measure 2 and 7, what is the hypotenuse?
- If the leg of a triangle is 5 and the hypotenuse is 10, what is the measure of the other leg?

High

- Explain the steps of solving a triangle given two legs and no hypotenuse and a triangle with one leg and one hypotenuse.
- Please solve this real world world problem using the Pythagorean theorem.

Cassie, a billionaire, was building a mall. Unfortunately, her contractors told her that she would need to remove the Abercrombie and Fitch store she had in her mall because the square roof wouldn't cover it. So, the brilliant Cassie decided to make her roof triangular. She found out that one of the sides needed to be 5 square feet. The hypotenuse 13 square feet. How many square feet must the final side of the triangular roof be in order to have Abercrombie and Fitch in Cassie's new mall?

- Create four of your own Pythagorean triples.
- Do you think the Pythagorean Theorem is useful? Why or why not?

6. Practice:

- A. Students will work on textbook material with a partner. (Check your understanding from textbook (stated in materials) is attached).
- B. Students will take a self-check quiz online to check their understanding of the concept. They will go over the quiz with the teacher individually after they complete the quiz. (The quiz accompanies the textbook)

7. **Create:** Students will create a poster on one of the many different Pythagorean Theorem proofs. The poster will require the proof and how to derive it. (Students will be given a list of creditable websites to use to find a different proof).

Websites:

- 1) <http://www.cut-the-knot.org/pythagoras/index.shtml>
- 2) <http://jwilson.coe.uga.edu/EMT668/emt668.student.fothers/HeadAngela/essay1/Pythagorean.html>
- 3) <http://www.ies.co.jp/math/java/geo/pythagoras.html>
- 4) <http://www.jimloy.com/geometry/pythag.htm>

8. **Presentations/Closure:** Students will present their proof to the class. They will work the proof out on the board for the students. The posters will be hung up around the room.

9. **Evaluation:** Rubric

Graded Task:

- Student used a credible website.
- Student wrote and derived the proof on a piece of poster paper.
(Supplied)
- Student correctly completed the proof on the poster.
- Student was able to present the proof correctly and clearly.

<i>Category</i>	<i>Description</i>	<i>Points</i>
No Attempt	The student does not make any attempt to do the assignment	0
Below Average	The student makes an attempt to complete the assignment. The student does not use an appropriate website. The proof is inaccurate and not sensible.	1
Average	The student selects an appropriate website. The student attempts the proof, yet shows limited understanding of the mathematical concepts. The proof contains some errors.	2
Satisfactory	The student uses an appropriate website. The student demonstrates substantial understanding of the proof. Student meets the requirements of the assignment but the proof contains minor errors and the explanation is unclear.	3
Excellent	The student uses a credible website. The student demonstrates a thorough and complete understanding of the proof. A complete response with a detailed explanation is provided. The student correctly uses mathematical procedures. The Students proof is solved and presented correctly and clearly.	4

10. **Accommodations:**
- A. Hearing Impaired:** Give the student preferential seating to where the teacher will be centrally speaking from. Speak loudly, to the student, and repeat as necessary. Allow the students to have a hard copy of the notes and put everything the teacher says on the board.
 - B. Visually Impaired:** Allow students to have preferential seating to best see any visual aids used. Allow student to have a tape-recorder. Make sure notes are big and bold and the lines on the paper are big. Make sure student has enough, or extra time.
 - C. Physically Impaired:** Have student work with a partner.
 - D. Gifted:** Have more challenging problems for the higher level students.
 - E. ESL:** Have a printed set of instruction in alternative language of give a traditional English set to any ESL assistants ahead of time.
11. **Role of Auxiliary Personnel:** If auxiliary personnel are available have them walk around and help during group work or individual work while students are at their seats.
12. **Family/Community:** The teacher will send home a flyer telling the family/community that the students learned about right triangles and the Pythagorean theorem today in class. The flyer will say that the students should explain to their family/community what a right triangle is. The students are then expected to find right triangles around their house with someone in their family/community. The flyer will also ask the students to make a list of their findings.

13. **Reflection/Self Evaluation:**

- A. Did I manage my time effectively?
- B. Did I represent the concept in enough ways to reach the students?
- C. Was my motivational activity effective?
- D. Was there anything I wasn't prepared for?
- E. What could I have done differently to improve this lesson?